Resonant Modes, Acoustic Impedance, and Surface Vibration Profiles of Periodic Piezoceramic Polymer Composite Plates

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Abstract—In this paper, the results of a combined analytical and experimental investigation of many properties, such as the resonant modes, the acoustic impedance, and the surface vibration profile of a periodic piezocomposite plate under different boundary conditions (both in air and in fluid), are presented. Due to the approach in the analysis model, we can quantitatively predict how various modes and their coupling to the thickness resonance change as the aspect ratio of the ceramic plate changes. It is also shown that the input acoustic impedance between a fluid or a solid medium and a composite is dispersive and depends on the medium. These findings will have important implications to the design of the composite, matching layer and backing for a composite ultrasonic transducer.

I. INTRODUCTION

For composites with 1-3 and 2-2 connectivities, both analytical and finite element modeling have been carried out and the results have provided useful guidelines in the design of a composite transducer. The isostrain model developed by Smith et al.[1] linked the material parameters of the constituents to the effective electromechanical properties of 1-3 and 2-2 composites respectively, which are in good agreement with experiments for composite with high aspect ratio t/d (where t is the thickness and d is the periodicity of the composite). Auld et al.,[2] using the Floquet theory, studied wave propagation in 2-2 and 1-3 composites and showed that because of the periodic structure of these composite materials, there exist pass bands and stop bands, similar to the band structure in a crystal solid, and the piezoelectric resonances associated with the stop band edge resonance. But the results are only in qualitatively agreement with experiments. In order to predict the frequencies of the lateral modes in a 2-2 composite, Wang et al.[3], Alippi et al.[4] and Oakley[5] investigated the wave propagation in one dimensional composites. It is shown that to some extent, the lateral modes can be approximately predicted either by shear wave or by both longitudinal wave and shear wave as well as Lamb wave propagation depending on the ceramic volume fraction in a composite. The mode coupling between the lateral modes and thickness mode was studied by Cracium et al.[6] using the coupling theory.

Recently, we developed an analytical model on the dynamic problem of a piezocomposite material with 2-2 connectivity.[7,8] In the model, we avoided the approximations made in the earlier analytical works and hence, can address the dynamic response of a 2-2 piezocomposite, such as the frequencies of various modes, the modes coupling and the electromechanical coupling coefficient, etc., in a more realistic and consistent manner.

In this paper, we will first sketch the analytical model for a 2-2 composite, and then give the theoretical results and their experimental comparisons.

II. THE SKETCH OF THE ANALYTICAL MODEL

Shown in Fig. 1 is a schematic drawing of a 2-2 composite, where the piezoceramic and the polymer plates form a parallel array. The coordinate system is chosen such that the x_3-axis is along the ceramic poling direction, the x_1-axis is perpendicular to the interface between ceramic and polymer plates, and the x_2-axis is in the plane of the plates. For a typical 2-2 composite, the dimensions in the x_1- and x_2-directions are much larger than the period d and the thickness t. So that, they can be taken as infinite without much error in the results. Under these conditions, the composite is clamped in the x_2- direction so that the strain S_2 is zero, and the problem therefore becomes a two dimensional one which is not dependent on the x_2-coordinate. The governing equations for the dynamics of a 2-2 composite are

$$T_{ij,i} = \rho \ddot{u}_i \quad (1)$$

$$D_{i,i} = 0 \quad (2)$$

where T is the stress, u the elastic displacement, and D the electric displacement.

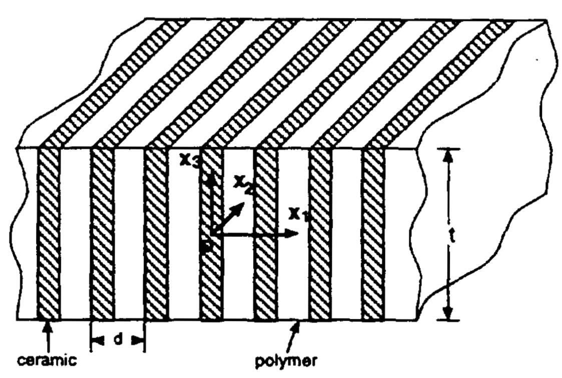

Fig. 1. Schematic drawing of a 2-2 piezocomposite.

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The constitutive equations for ceramic phase are as follows

$$[T]=[c^E][S]-[e_t][E] \tag{3}$$

$$[D]=[e][S]+[\varepsilon^S][E] \tag{4}$$

where E is the electric field, c the stiffness, e the piezoelectric coefficient and ε the dielectric permitivity, respectively. Here $[e_t]$ is the transposed $[e]$ array. The wave propagation solutions in a 2-2 composite are

$$\begin{aligned} u_3 &= A\exp(j(hx_1+\beta x_3-\omega t)) \\ u_1 &= B\exp(j(hx_1+\beta x_3-\omega t)) \\ \Phi &= C\exp(j(hx_1+\beta x_3-\omega t)) \end{aligned} \tag{5}$$

where A, B, and C are constants, ω is the angular frequency. h and β are the wave vector components in the x_1- and x_3-directions, respectively.

For the polymer, we have the same equations as the ceramic except that the piezoelectric coefficients are zero.

From above equations, we can obtain three partial waves for both ceramic and polymer plates. In the ceramic plate, they are quasi-shear wave, quasi-longitudinal wave and quasi-electrical wave. In polymer plate, they are pure shear wave, pure longitudinal wave and static electric wave. Superpositin of these partial waves in both ceramic and polymer plates yields the expressions of the displacements and the stresses. When the interface boundary conditions are exactly satisfied, symmetric and anti-symmetric modes can be obtained, but only the symmetric modes are piezoelectric active in a completely electroded 2-2 composite plate.

In order to solve the vibration problem of a 2-2 composite plate, the displacements are expanded in terms of the eigenfunctions in an unbounded system. Variational method is used to satisfy the boundary conditions. From here, all the properties of a 2-2 composite, such as, resonant modes, acoustic impedance, vibration displacement profile and electromechanical coupling coefficient, etc., can be evaluated.

III. Results and discussions

A. *Dispersion Curves*

Fig. 2 shows the dispersion curves of 2-2 composites in which the ceramic volume fraction is 44% and 80% respectively. At the same time, the dispersion curves of both the single ceramic plate under stress free boundary condition and the single polymer plate under fixed displacements boundary condition, which corresponds approximately to the situation of the ceramic and the polymer plates in a 2-2 composite, are shown in Fig. 2 too. From here, it is found that the dispersion curves of a 2-2 composite are related to those of the single ceramic and the polymer plates. At small βd, the first branch represents the longitudinal wave propagating in x_3-direction, and the second branch of the composite with low ceramic volume fraction corresponds to the lateral resonance of the polymer plate which is mainly determined by the shear wave velocity of the polymer. However, for higher volume fraction, such as 80%, the second branch exhibits the transverse vibration of the ceramic plate due to the piezoelectric coefficient d_{31}. That is the reason why one can approximately predict the lateral modes using the shear or the longitudinal wave propagation along the x_1-direction.

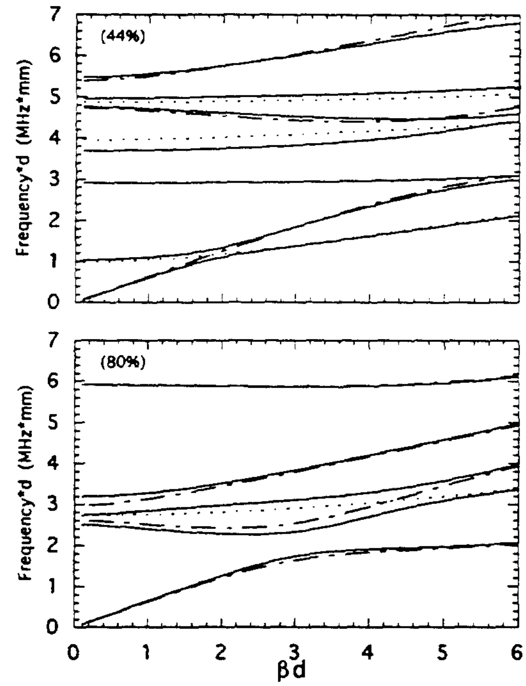

Fig. 2. Dispersion curves of 2-2 composite. The solid lines are for composite, the dashed lines for single ceramic plate, dotted lines for single polymer plate.

B. *Resonance Modes*

The free vibration of a finite composite plate driven by a pulse electric field exhibits a lot of modes. Except for contour extensional resonant modes, we can predict all the other modes which are related to the thickness resonance and the lateral resonance. Fig. 3 shows the electric impedance curve of the 44% 2-2 composite with a aspect ratio t/d equal to 4. Except for the thickness mode and its third harmonic, there appears another two modes called the first and the second lateral modes.

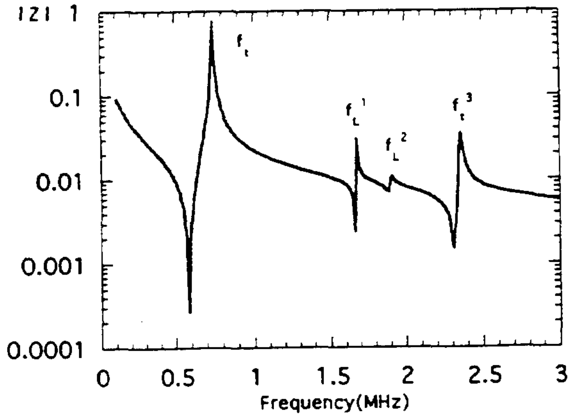

Fig. 3. The electric impedance spectrum of a 44% 2-2 composite with t/d=4.

By examining the equations of the boundary conditions at x3=±t/2, it can be deduced that a resonance will occur whenever $\beta=(1+2n)\pi/t$, i.e., $\cos(\beta t/2)=0$. From the dispersion curves, it is clear that the fundamental thickness resonance and the first lateral resonance occur at $\beta=\pi/t$. Similarly, when $\beta=3\pi/t$, the second lateral mode and the third harmonic of the thickness mode will show up at the first and the second branches respectively. As the aspect ratio decreases, the coupling between the thickness mode and the first lateral mode becomes stronger. As the aspect ratio is further decreased, the second lateral mode will disappear in the 44% 2-2 composite. In fact, whether this mode shows up or not depends on the electromechanical coupling coefficient of that mode.

Table I shows the theoretical and experimental resonant frequencies of a 80% 2-2 composite plate with the aspect ratio t/d of 1.304.

From Table I, it is found that the modes structure of the 80% 2-2 composite plate is much more complicate than that of the 44% 2-2 composite plate because the transverse vibration of the ceramic plate has larger electromechanical coupling coefficient so that the third, the fifth, even the seventh harmonics will show up. The theoretical results are in excellent agreement with the experimental results except the last column. The probable reason for this error is that the parameters of the polymer phase to be used in the calculation have larger error because these two frequencies are related to the modes of the polymer plate in the composite.

TABLE I
THE RESONANT FREQUENCIES OF 80% 2-2 COMPOSITE PLATE

cal(MHz)	0.8496	1.0800	1.2345	1.2840	1.6667
exp(MHz)	0.8309	1.0446	1.2360	1.3175	1.4938
cal(MHz)	1.8000	2.0551	2.4522	2.5335	2.6665
exp(MHz)	1.8016	2.0976	2.4612	2.5630	2.8675

C. *Electromechanical Coupling Coefficient*

For piezo-materials, the electromechanical coupling coefficient is a key parameter, especially for transducer applications. According to the definition of IEEE,[9] it can be evaluated by

$$k_t^2 = \frac{\pi}{2}\frac{f_s}{f_p}\tan\left(\frac{\pi}{2}\frac{f_p - f_s}{f_p}\right) \tag{6}$$

where k_t is the thickness mode coupling factor, f_s and f_p are the series and the parallel resonant frequencies respectively.

Fig. 4 is the electromechanical coupling coefficient of the first and the second mode vs. d/t for 2-2 composite with 44% ceramic volume fraction. From here, it is found that the coupling factor of the first mode decrease with d/t. but that of the second mode increases. That is why the second lateral mode disappears at small d/t.

D. *Acoustic Impedance*

Unlike single phase piezo-material, a 2-2 piezoceramic-polymer material is nonuniform, so that it is dispersive. In order to investigate the input acoustic impedance at a interface between a 2-2 composite and a medium, the plane wave reflection from that interface is studied.[10] The input acoustic impedance can be evaluated by

$$Z_{in} = \frac{1-R}{1+R} Z_f \tag{7}$$

where Z_{in} is the input acoustic impedance of the 2-2 composite, Z_f the acoustic impedance of the medium, and R the reflection coefficient. The input acoustic impedance can also be evaluated by

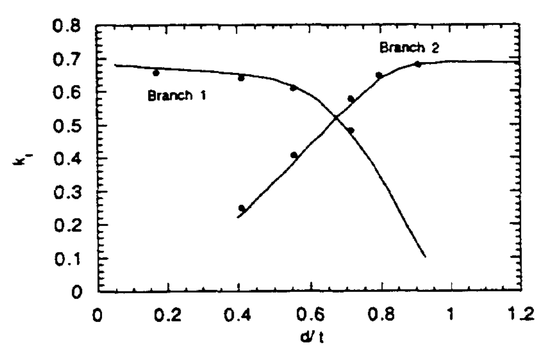

Fig. 4. The electromechanical coupling coefficient changes with d/t for the first and the second modes. The dots are the experimental data.

$$Z_{in} = \frac{\overline{T}}{\overline{V}} \tag{8}$$

where $\overline{T}$ and $\overline{V}$ are the averages of stress T and particle velocity V.

The theoretical results show that the input acoustic impedance of a 2-2 composite is complex, the magnitude decreases with frequency when the frequency is below the first lateral mode , but the phase increases. In addition, it is a function of the impedance of the medium. These results are shown in Fig. 5.

E. *Surface Vibration Profile*

The acoustic field radiated by a transducer is determined by the displacement distribution on the front face of the transducer. In the practical composite materials, this distribution is not uniform because it is difficult to make the aspect ratio t/d very large. Hence, it is important to study the effects of the aspect ratio on the surface profile.

It is found that for a given t/d, the ratio of the surface displacement in the center of the polymer plate and in the center of the ceramic plate increases with frequency when the frequency is below the first lateral mode. At some frequency f_1, which is close to resonance frequency f_t , this ratio is one. We define the frequency range in which this ratio changes from 0.9 to 1.1 as the bandwidth Δf, which affects the operating frequency range of a broadband transducer. The changes of f_1/f_t and $\Delta f/f_1$ with aspect ratio t/d of a 44% 2-2 composite is shown in Fig. 6.

IV. Conclusion

In this paper, it was shown that the electromechanical properties of a 2-2 composite can be exactly modeled by the analytical model. The resonance modes in a composite plate can be predicted by using the dispersion curves which related to that of the single ceramic and polymer plate under the appropriate boundary conditions. It was also shown that the acoustic impedance of the 2-2 composite is related to the medium.

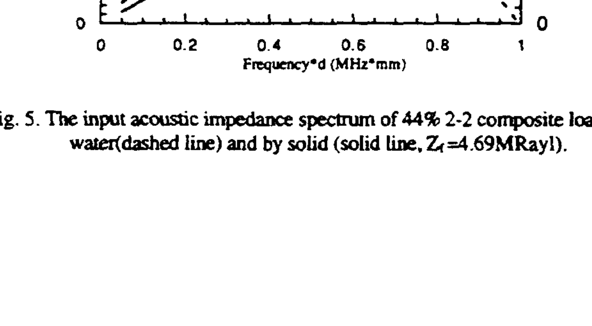

Fig. 5. The input acoustic impedance spectrum of 44% 2-2 composite loaded by water(dashed line) and by solid (solid line, Z_f=4.69MRayl).

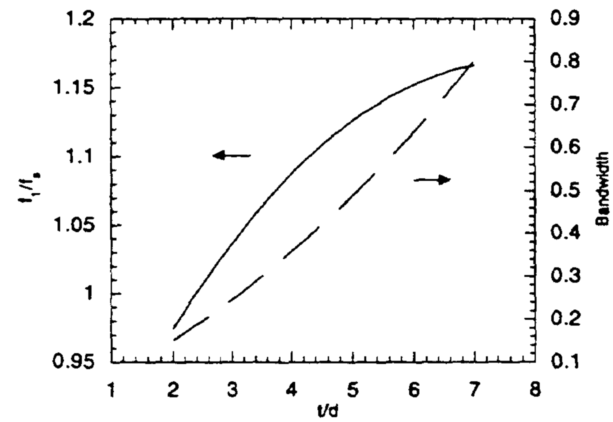

Fig. 6. The changes of f_1/f_t and $\Delta f/f_1$ with aspect ratio t/d of a 2-2 composite with 44% ceramic volume fraction.

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